

## Analytical Solution for Heat Transfer in He II High Reynolds Number Flow

Baudouy, B., NHMFL

Van Sciver, S.W., NHMFL/FAMU-FSU College of Engineering

A model has been developed to represent heat transfer in He II flow with a bulk velocity of order that of second sound (~20 m/s) and Reynolds number to  $10^7$ .<sup>1</sup> At these velocities, the Joule-Thomson effect is not negligible compared to other transport processes as diffusion and convection. The Joule-Thomson effect will create a 33 mK/m temperature gradient for a bulk velocity of 18.5 m/s at 1.9 K. The Joule-Thomson coefficient is defined as  $(\partial T / \partial p)_h = (\alpha T - 1) / \rho C_p$ , where  $T$  is the temperature,  $p$  the pressure,  $\rho$  the density,  $C_p$  the specific heat at constant pressure, and  $\alpha$  the coefficient of thermal expansion. It modifies the heat flux in flowing He II to

$$q = \left[ f^{-1} \left( \frac{\nabla p}{\rho S} - \nabla T \right) \right]^{1/2}$$

where  $S$  is the entropy and  $f^{-1}$  is the heat conductivity. In steady state, the energy equation is also modified as,

$$\nabla \left[ f^{-1} \left( \frac{\nabla p}{\rho S} - \nabla T \right) \right]^{1/2} + \rho v C_p \nabla T + v \nabla p = 0$$

Using ad hoc variable change in one dimension, and considering the pressure gradient constant, this equation has an analytic solution. Figure 1 presents the solution as a function of the velocity for fixed temperature at the boundary. One can notice that because of the Joule-Thomson effect, the temperature can have a maximum at a location other than the boundaries. Another important result is that for velocity higher than 5 m/s, for a 1 m long channel, the diffusion is negligible compared to convection. This point is demonstrated for the highest velocities by the temperature gradient being positive, and almost constant over the entire channel except very near the "cold" boundary.

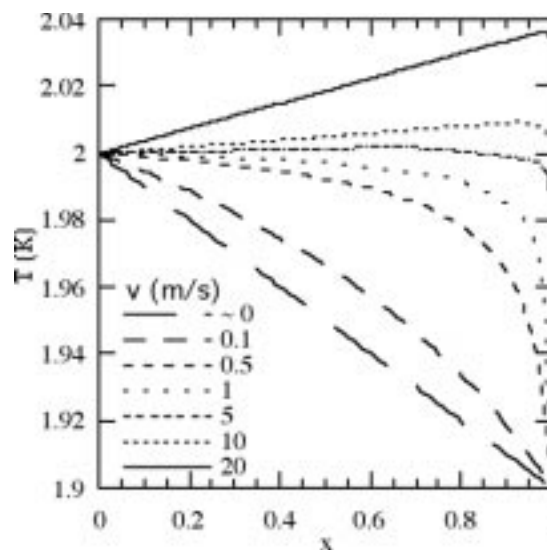


Figure 1. Temperature profile as a function of the velocity.

Reference:

- 1 Walstrom, P. L., *Cryogenics*, **28**, March (1988).

## Fluid Dynamics in Two-Phase Helium II

Panek, J., NHMFL/FAMU-FSU College of Engineering  
Van Sciver, S.W., NHMFL/FAMU-FSU College of Engineering

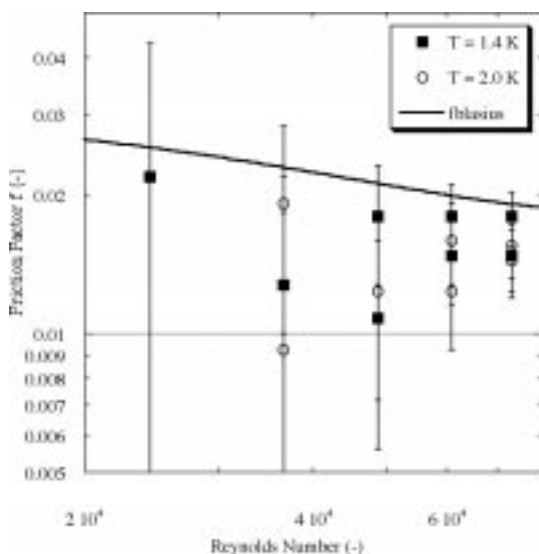
We report on horizontal two-phase He II fluid dynamics and vapors in a 2 meter long test section with cross section 3 mm wide and 65 mm tall. Bellows pumps at each end of the channel add or remove liquid at up to 2 g/s (30 cm/s). Liquid levels are measured to within  $\pm 1$  mm with capacitive probes using a tunnel diode-based cold oscillator circuit. The liquid height change across the channel is measured as a function of the mass flow rate.

To understand this system, a simple model was developed based on force balances over a control volume of liquid. The result is an expression for the friction factor,  $K$ , as a function of the measured liquid levels at the two ends of the channel,  $y_1$  and  $y_2$ ,

$$K = \frac{\rho g w (y_1^2 - y_2^2) - 2m(v_1 - v_2)}{\rho \bar{w}^2 PL}$$

where  $w$  is the channel height,  $P$  is the wetted perimeter, and  $L$  is the length. Calculations based on this model are compared to the isothermal experimental results (Figure 1). The measured friction factors agree well with the classical smooth tube Blasius correlation.

A second data set for non-isothermal flow is presented. Height changes as a function of mass flow rate are asymmetrical, and much higher in magnitude than for the isothermal case. Given the friction factor calculated from the previous result, it is possible to derive the necessary change in saturated pressure at the liquid-vapor interface, and express it in terms of a saturated temperature change.



**Figure 1.** Isothermal two-phase forced flow friction factor vs. Reynolds number.

## Heat Transfer in Horizontal Two-Phase Helium II

Panek, J.S., NHMFL/FAMU-FSU College of Engineering  
Van Sciver, S.W., NHMFL/FAMU-FSU College of Engineering

We report on heat transport in stratified horizontal two-phase He II and vapor in a 2 m long test section with a cross section 3 mm wide and 65 mm tall. Liquid and vapor temperatures are measured at eight

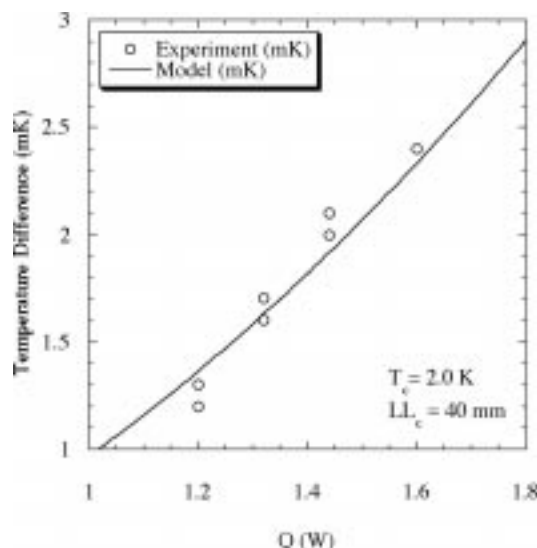
locations, six along the channel and two in the boundary reservoirs. The six measurements along the channel are at three equally spaced axial locations, each with one thermometer in the liquid, and one in the vapor. Boundary temperatures selected are 1.4 K, 1.8 K, and 2.0 K, and up to a 20 mK temperature difference across the channel. The liquid level at the cold end of the channel is set at 60 mm, 50 mm, or 40 mm, and the warm end level is lower due to differences in saturated pressure. The effect of forced flow up to 0.66 g/s (10 cm/s) of liquid with or against the temperature gradient is also measured.

A numerical model is constructed that solves the main governing nonlinear ordinary differential equation for energy transport,

$$A(T)\left(\frac{dT}{dx}\right)^{1/2} + B(T)\left(\frac{dT}{dx}\right)^{1/3} = Q$$

This model assumes that the two main heat transport mechanisms are due to vapor mass flow and counterflow in the almost static liquid. The terms  $A(T)$  and  $B(T)$  are functions of the local vapor and liquid fractions. The results of the analysis are compared with the experiment. Agreement is good if one assumes a certain reasonable value for parasitic heat load to the experiment (Figure 1).

The main conclusions from the work are as follows.  
(1) A simple one-dimensional numerical model can



**Figure 1.** Temperature difference across the 2 m long channel versus applied heat flux. Data (○) are compared to the numerical model (—) with an assumed background heat leak of 1.1 W.

predict the heat and mass transport in stratified two-phase He II flow. The two dominant energy transport mechanisms working in parallel are turbulent heat transport in the liquid, according to the two fluid models, and enthalpy flow in the vapor. (2) The liquid level change due to temperature is represented to within 10% by a hydrostatic model based on the saturated pressure change. (3) The temperature profile in a two-phase channel shows a flattening in the center of the channel with respect to the ends, suggesting some type of entrance and exit loss mechanism. (4) Forcing the liquid to flow with or against the temperature gradient does not affect the temperature profile within the measurement accuracy for the flowrates used in this work, 0.66 g/s (liquid velocity 10 cm/s). (5) A tall (65 mm) and thin (3 mm) channel does not unduly impede mass transport between the vapor and liquid despite the large aspect ratio of 22:1 and temperature gradients of up to 10 mK/m. If any superheating is present in the vapor, it is less than a few mK.

## Observed Drag Crisis on a Sphere in Flowing Helium I and Helium II

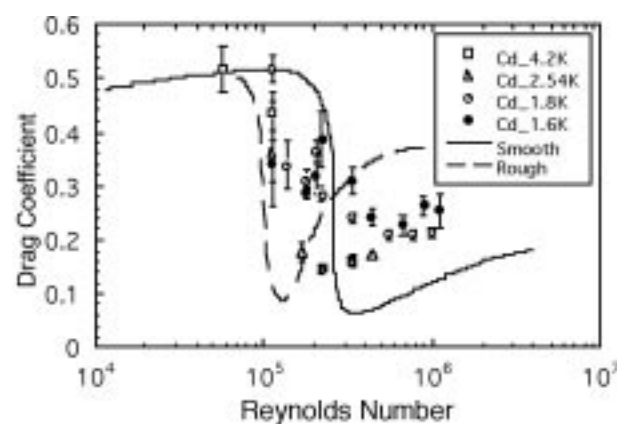
Smith, M.R., NHMFL

Hilton, D.K., NHMFL

Van Sciver, S.W., NHMFL/FAMU-FSU College of Engineering

We have measured the pressure distribution on the surface of a 10 mm sphere in flowing helium I and helium II as a function of Reynolds number (dimensionless velocity).<sup>1</sup> The pressure distribution was measured on a meridian, between the upstream stagnation point and the point directly downstream. A single pressure tap was used, and the sphere rotated with respect to the flow upon a support strut oriented perpendicular to the flow. One objective was to observe the transition from laminar to turbulent boundary layer. In both helium I and helium II, we have seen many instances of pressure profiles that are typically associated with turbulent boundary layers. Additionally, there are a few profiles that suggest transitional behavior at the lower Reynolds numbers. By integrating the pressure distributions, assuming azimuthal symmetry of the flow field, and minimal

interference from support strut and flow boundaries, we have calculated the drag as a function of Reynolds number. Drag coefficients (drag expressed in dimensionless form) are plotted in Figure 1 for both helium I and helium II against accepted classical correlations<sup>2</sup> for both smooth and non-smooth spheres. Error bars were calculated based upon the statistical scatter in the data making up the pressure profiles.



**Figure 1.** Drag coefficient vs. Reynolds number. Open squares and triangles correspond to temperatures 4.2 K and 2.54 K, respectively. Dotted and closed circles were recorded at 1.8 K and 1.6 K, respectively (helium II phase). The solid line<sup>2</sup> represents commonly accepted drag crisis data, and the dashed<sup>2</sup> line shows the effect of a surface roughness of  $\epsilon/d=0.0015$ .

Results in helium I, above the superfluid transition, suggest good agreement with classical data. Further, data taken in helium I at different temperatures are in excellent agreement with one another. Latest results in helium II indicate that the drag crisis, associated with the transition from laminar to turbulent boundary layer, occurs at a Reynolds number of approximately  $2 \times 10^5$ , in fair agreement with classical data. Although drag crises are apparent in both helium I and helium II, the slight shift in the data between helium I and helium II, or between two temperatures in helium II, suggests that temperature may play a role in helium II fluid dynamics. Further study to determine the nature of this temperature dependence is clearly warranted.

### References:

- <sup>1</sup> Smith, M.R., *et al.*, to be published in *Physics of Fluids*.
- <sup>2</sup> Achenbach, E., *J. Fluid Mech.*, **65**, 113 (1974).